27. J. W. Rudnicki and J. R. Rice, "Condition for the localization of deformation in pressure-sensitive dilatant materials," J. Mech. Phys. Solids, 23, No. 6 (1975).
28. L. V. Nikitin and E. I. Ryzhak, "Fracture of rock with internal friction and dilatation," Dokl. Akad. Nauk SSSR, 230, No. 5 (1976).
29. V. N. Nikolaevskii, "Determination of the equation of plastic deformation of a granulated medium," Prikl. Mat. Mekh., 35, No. 6 (1971).
30. I. A. Garagash, "Stability and fracture of rock masses with applications in the mechanics of earthquake preparation," Author's Abstract of Doctor of Physico-Mathematical Dissertation, IFZ Akad. Nauk SSSR, Moscow (1985).
31. Ya. Rykhlevskii, "On Hooke's law," Prikl. Mat. Mekh., 48, No. 3 (1984).
32. A. I. Chanyshev, "On the plasticity of anisotropic bodies," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1984).
33. N. I. Muskhelishvili, Some Basic Problems of the Mathematical Theory of Elasticity [in Russian], Izd. Akad. Nauk SSSR, Moscow (1949).
34. D. D. Ivlev, Theory of Ideal Plasticity [in Russian], Nauka, Moscow (1966).
35. A. M. Kovrizhnykh, "Variant of the theory of plastic deformation of rocks," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 1 (1983).
36. N. I. Bezukhov, Principles of the Theory of Elasticity, Plasticity, and Creep [in Russian], Vysshaya Shkola, Moscow (1968).

CRACK GROWTH IN METALS AT ELEVATED TEMPERATURE

A. G. Cherepanov and G. P. Cherepanov

UDC 539.375+539.376

Creep is the capacity of all solids to undergo irreversible deformation under constant loads due to the thermal motion and directional migration under load of the main sources of such deformation: inclusions, voids, dislocations, and microcracks. The latter, coalescing at the final stage of creep, form a macrocrack which separates the structural element. Creep in metals usually becomes noticeable at temperatures greater than one-third of the melting point (in K).

The phenomenological approach to creep is semi-empirical and is based on many additional assumptions regarding irreversible (plastic) strains that have been justified on the basis of experiments for specific materials under certain conditions [1].

The new approach being taken to fracture mechanics in creep and plasticity consists of the following: the material is considered to be linearly or nonlinearly elastic, while the sources of irreversible strain are examined in explicit form [2, 3]. In this approach, irreversible strain is calculated as being the result of the nucleation, movement, and growth of these sources, while fracture is represented by a certain calculable critical moment of instability of plastic strain. It is possible to examine different deterministic and statistical systems of sources by using the methods of the theory of diffusion and migration to study their motion and development [2, 3].

Since the 1970s and the publication of [4], the growth of creep cracks in metals has been subjected to massive experimental study within the framework of classical fracture mechanics on the basis of stress intensity factors [5] and invariant energy integrals [6-18].

As was shown in [19], the δ_K-concept in the Leonov—Panasyuk—Dugdale model follows from the general energy-based Γ_c-concept. The analog of the δ_K-concept for linear viscoelastic materials was developed in [20]. In this case, the Γ_c- and δ_K-concepts differ.

In the theory of elasticity, invariant integrals were first found by the Maxwell method by Eshelby in 1951. The basic invariant energy integral (more general than Eshelby's) used as a criterion in the theory of fracture was obtained directly from the conservation law in [6] for an arbitrary solid. Obtained with it was the solution of the problem

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 144-150, September-October, 1989. Original article submitted September 29, 1987; revision submitted January 19, 1988.

of the stress and strain distribution near the end of a crack in an incompressible nonlinearly elastic body described by a power law (this solution was also later obtained in [25, 26]). The latter solution was used to analyze creep cracks in [10]. In [7], the Eshelby integral was employed to calculate the stress and strain concentration in notches. The authors of [9] used the invariant integral as a criterion in the theory of fracture. In [21-24], the integral was used to calculate the energy flux at the end of a dynamic moving crack.

As a parameter controlling the growth of a creep crack, experimenters have tried the following parameters: stress intensity factor K_I; crack opening δ; rate of opening $\dot{\delta}$; invariant integrals J and C*; the mean stress in the net cross section σ_{net}, etc. [10-18]. Investigators have found specific functional correlations between the crack growth rate $\dot{\ell}$ and various other parameters, but no general law has yet been established. Even the question of the best fracture parameter essentially remains open. In our opinion, the most interesting empirical result is the conclusion regarding the approximate linearity of the relations $\dot{\ell} \sim \dot{\delta}$, $\dot{\ell} \sim C^*$ seen in several studies in a certain range of loads. We should also point out the studies [27-36] on creep cracks. It can be justifiably asserted that there is currently a crisis in this area of study, this situation being the result of two main factors.

1. Difficulties in setting up conclusive experiments involving the growth of creep cracks. In fact, subcritical crack growth in metals is in most cases due to chemical or electrochemical reactions occurring at the end of a crack with the participation of the metal and the environment [11]. Thus, strictly speaking, creep tests should be conducted in a vacuum or inert gas (with special chemical monitoring); this condition has been considered only in a few studies. Furthermore, even an inert medium does not guarantee that the reason for subcritical crack growth is creep of the metal, since the latter may be due to atomic hydrogen dissolved in the metal's lattice [11]. Thus, it is also necessary to carefully check for hydrogen in the lattice.

2. Difficulties in formulating a sufficiently simple and general phenomenological theory of creep and plasticity. The main problem is that classical creep theories are satisfactory for large values of τ and T but only for low values of p (where T is temperature, p is the characteristic stress, and τ is the loading time), while classical theories of plasticity are satisfactory at high p but low τ and T. The region of the tip of a creep crack is characterized by high p and T within a broad range of τ. Thus, a combined but sufficiently simple theory of plasticity and creep is needed in this region.

The most promising method of resolving this crisis is to avoid semi-empirical theories of fracture and to change over to theories based only on fundamental atomic and microscopic constants. This is the quantum-mechanical approach to fracture proposed in [37-40]. In accordance with the quantum mechanics of fracture, the material is considered to be a linearly elastic body with a characteristic crystalline structure, while the irreversible plastic strains and creep and the process of creep-crack growth itself (i.e., the fracture process) is calculated on the basis of analysis of the nucleation and movement of microcracks, dislocations, inclusions, and voids due to thermal fluctuations.

Below, we examine a simple phenomenological approach to the growth of a creep crack which is based on a combination theory of plasticity and creep.

Special Theory of Plasticity and Creep. Let a homogeneous isotropic medium have the properties of an incompressible, nonlinearly elastic, power-law body and an incompressible, nonlinearly viscous, power-law fluid. Thus,

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{v}, \quad \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \varepsilon_{ij}^{v}; \tag{1}$$

$$\varepsilon_{ij}^{e} = 0, \quad \varepsilon_{ij}^{v} = 0; \tag{2}$$

the governing relations for the elastic component $\varepsilon_{ij}{}^{e}$:

$$\varepsilon_{ij}^{e} = \frac{\varepsilon_s}{\sigma_s}\left(\frac{J}{\sigma_s}\right)^{\varkappa}\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right), \quad D^{e} = \varepsilon_s\left(\frac{J}{\sigma_s}\right)^{\varkappa+1}, \tag{3}$$

$$(D^{e})^2 = \frac{2}{3}\left(\varepsilon_{ij}^{e} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}\right)\left(\varepsilon_{ij}^{e} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}\right),$$

$$J^2 = \frac{3}{2}\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right)\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right);$$

while the governing relations for the viscous component ε_{ij}^{v}:

$$\dot{\varepsilon}_{ij}^{v} = \frac{\gamma_s}{\tau_s}\left(\frac{J}{\tau_s}\right)^{\lambda}\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right), \quad \dot{D}^{v} = \gamma_s\left(\frac{J}{\tau_s}\right)^{\lambda+1}, \tag{4}$$

$$(\dot{D}^{v})^2 = \frac{2}{3}\left(\dot{\varepsilon}_{ij}^{v} - \frac{1}{3}\dot{\varepsilon}_{kk}^{v}\delta_{ij}\right)\left(\dot{\varepsilon}_{ij}^{v} - \frac{1}{3}\dot{\varepsilon}_{kk}^{v}\delta_{ij}\right).$$

Here ε_{ij} are the strains; σ_{ij} are the stresses; $\dot{\varepsilon}_{ij}$ is the strain rate; D^e, J, and $\dot{D}^v$ are the intensity of the elastic strains and stresses and the rates of viscous deformation; δ_{ij} is the Kronecker symbol; κ, λ, ε_s, σ_s, γ_s, τ_s are empirical constants chosen so as to obtain the best approximation of the plasticity curve σ vs. ε (with different assigned $\dot{\varepsilon}$) and the creep curve $\dot{\varepsilon}$ vs. ε (with different assigned σ); the dots above the letters denote a time derivative. The range of variation of κ and λ from two to ten covers most of the metals and alloys encountered in practice. Since we have the relationships $\varepsilon_s\sigma_s$ = A = const, $\gamma_s\tau^{-\lambda-1}$ = B = const, there are altogether four independent parameters present in the creep-plasticity theory being examined.

Together with the equations of equilibrium and compatibility, Eqs. (1)-(4) constitute a closed system of equations of creep-plasticity theory. It follows from here that in the limiting cases $\gamma_s = 0$ and $\varepsilon_s = 0$, we obtain power-law plasticity and power-law creep, respectively.

The following obvious analogy is valid: if only the loads at the boundary of the body are assigned, the stress distribution for $\gamma_s = 0$ will be identical to the distribution for $\varepsilon_s = 0$.

It should be noted that the proposed special theory of plasticity-creep is distinguished by its simplicity and the fact that it is general enough for many practical applications. The general plasticity-creep theory is described by a time-dependent functional F of the stress tensor $\widehat{\sigma}$, strain tensor $\widehat{\varepsilon}$, and strain-rate tensor $\widehat{\dot{\varepsilon}}$:

$$F(\widehat{\sigma}, \widehat{\varepsilon}, \widehat{\dot{\varepsilon}}) = 0. \tag{5}$$

In the simplest case, this corresponds to the hypothesis of an equation of state [1].

Neighborhood of the Tip of a Moving Crack. We will examine the stress and strain distribution in the small region of the tip of a normal-rupture crack which is steadily growing at a constant rate along the x axis in the medium being considered. Here, a dot above a letter denotes the operator $V\partial/\partial x$, where V is the crack growth rate. The coordinate x is reckoned from the crack tip, which can be considered semi-infinite. In polar coordinates $r\theta$ with their origin at the crack tip, the main equations of the special theory of plasticity-creep take the following form:

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \quad \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + 2\frac{\tau_{r\theta}}{r} = 0; \tag{6}$$

while the strain compatibility condition

$$2\frac{\partial}{\partial r}\left(r\frac{\partial\varepsilon_{r\theta}}{\partial\theta}\right) = \frac{\partial^2\varepsilon_r}{\partial\theta^2} - r\frac{\partial\varepsilon_r}{\partial r} + r\frac{\partial^2(r\varepsilon_\theta)}{\partial r^2}; \tag{7}$$

and the relations between the strains, stresses, and strain rates (plane strain)

$$\varepsilon_r = -\varepsilon_\theta = \varepsilon^v + \frac{1}{2}\frac{\varepsilon_s}{\sigma_s}\left(\frac{J}{\sigma_s}\right)^{\varkappa}(\sigma_r - \sigma_\theta) \quad (\varkappa \geqslant 0), \tag{8}$$

$$\varepsilon_{r\theta} = \varepsilon_{r\theta}^v + \frac{\varepsilon_s}{\sigma_s}\left(\frac{J}{\sigma_s}\right)^{\varkappa}\tau_{r\theta};$$

$$\dot{\varepsilon}_r^v = -\dot{\varepsilon}_\theta^v = \dot{\varepsilon}^v = \frac{1}{2}(\gamma_s/\tau_s)(J/\tau_s)^{\lambda}(\sigma_r - \sigma_\theta), \tag{9}$$

$$\dot{\varepsilon}_{r\theta}^v = (\gamma_s/\tau_s)(J/\tau_s)^{\lambda}\tau_{r\theta} \quad (\lambda \geqslant 0),$$

where

$$J = \frac{1}{2}\sqrt{3}\sqrt{(\sigma_r - \sigma_\theta)^2 + 4\tau_{r\theta}^2}, \quad D^e = \sqrt{2}\sqrt{(\varepsilon_r^e)^2 + (\varepsilon_{r\theta}^e)^2}, \tag{10}$$

$$\dot{D}^v = \sqrt{2}\sqrt{(\dot{\varepsilon}_r^v)^2 + (\dot{\varepsilon}_{r\theta}^v)^2}, \quad D^e = \varepsilon_s(J/\sigma_s)^{\varkappa+1},$$

$$\dot{D}^v = \gamma_s(J/\tau_s)^{\lambda+1} \quad \left(\sigma_z = \frac{1}{2}(\sigma_r + \sigma_\theta), \ \varepsilon_z = 0\right).$$

In the steady-state case, $\dot{\varepsilon}_{ik} = V\partial\varepsilon_{ik}/\partial x$, i.e.

$$V\frac{\partial\varepsilon_r^v}{\partial x} = -V\frac{\partial\varepsilon_\theta^v}{\partial x} = \frac{1}{2}\,\frac{\gamma_s}{\tau_s}\left(\frac{J}{\tau_s}\right)^\lambda(\sigma_r-\sigma_\theta), \tag{11}$$

$$V\frac{\partial\varepsilon_{r\theta}^v}{\partial x} = \frac{\gamma_s}{\tau_s}\left(\frac{J}{\tau_s}\right)^\lambda\tau_{r\theta}\quad\left(\frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\,\frac{\partial}{\partial\theta}\right).$$

The subscripts r and θ denote components of the quantities in the directions r and θ, respectively.

Invariant Criterion of Local Fracture. For a steadily moving crack, the Γ-integral [6, 11] is an invariant parameter in the vicinity of the crack tip:

$$\Gamma = \oint_\Sigma (Un_x - \sigma_{ij}u_{i,x}n_j)\,d\Sigma\quad (i,\,j=1,\,2). \tag{12}$$

Here, u_i and σ_{ij} are the Cartesian coordinates of the components of the displacement vector and stress tensor (the subscript 1 coincides with the subscript x); n_j are components of the unit vector of an outward normal to the contour Σ enveloping the end of the crack; U is the strain energy:

$$U = \int\sigma_{ij}\,d\varepsilon_{ij} = \int_{-\infty}^{x}\sigma_{ij}\frac{d\varepsilon_{ij}}{dx}\,dx = \frac{\varkappa+1}{\varkappa+2}JD^e + \frac{1}{V}\int_{-\infty}^{x}J\dot{D}^v\,dx. \tag{13}$$

We emphasize that the J- and C*-integrals are invariant in the given case and cannot be used as fracture criteria (the J-integral is invariant only for elastic media, while the C*-integral is invariant only for viscous fluids with the velocity potential W, when $\dot{\varepsilon}_{ij} = \partial W/\partial\sigma_{ij}$ and when the crack is stationary).

Any number of other invariant Γ-integrals can easily be obtained from Eqs. (12) and (13) through the substitutions

$$u_i\to\frac{d^nu_i}{dx^n},\quad \varepsilon_{ij}\to\frac{d^n\varepsilon_{ij}}{dx^n}\quad\left(D^e\to\frac{d^nD^e}{dx^n},\ \dot{D}^v\to\frac{d^n\dot{D}^v}{dx^n}\right) \tag{14}$$

(n is any positive integer). Instead of d/dx in Eqs. (14), in the general case we can use differentiation with respect to time d/dt or the operation of variation (incremental increase) of the loading parameter: the invariance of the Γ-integrals remains [11]. The J- and C*-integrals coincide with the corresponding Γ-integrals in special cases. It should be remembered that the Γ-integrals remain invariant for any continuous media [6, 11].

If as Σ we choose a narrow rectangle along the x axis with the dimensions 2L × 2ε, then the first term in (12) disappears, while the second is simplified and replaced by $\sigma_{i2}u_{i,x}$. In the present case of plane strain and normal-rupture cracks, the integral (12) is reduced to the form

$$\Gamma = -2y\int_0^\pi\sigma_\theta\sin^{-2}\theta\,du_\theta\quad\text{at}\quad\frac{\varepsilon}{L}\to 0, \tag{15}$$

since at $y = \varepsilon = r\sin\theta$ $dx = d(r\cos\theta) = -yd\theta/\sin^2\theta$; Γ is the dissipation of energy on the formation of a unit surface of the crack. Since this value is finite and nonzero, the product $\sigma_\theta\partial u_\theta/\partial x$ should have a singularity on the order of r^{-1} in the neighborhood of the crack tip [6, 11]. The parameter Γ retains its value as an invariant criterion of local fracture also for cracks moving with an arbitrary variable velocity V(t) if the function $\dot{V}(t)$ is continuous. In this case, there is always a certain sufficiently small region of the crack tip in which the stress and strain field can be made to differ as little as desired from the steady-state case (local stationariness [11]). Here, the general law of crack growth is written as

$$V = G(\Gamma)\quad (V = \dot{l}).$$

In this equation, G(Γ) is a function determined experimentally or on the basis of structural theory.

Analysis of the Stress and Strain Field near the End of a Crack. The stress and strain field in the neighborhood of the tip of a crack moving in the medium being examined is determined from the solution of system (6)-(11) with homogeneous boundary conditions for the half-plane $0 < r < \infty$, $0 \le \theta \le \pi$:

$$\tau_{r\theta} = 0,\ \partial\sigma/\partial\theta = 0 \ \text{at}\ \theta = 0,\ \sigma_\theta = \tau_{r\theta} = 0 \ \text{at}\ \theta = \pi. \quad (16)$$

With $\theta = 0$, conditions (16) constitute the symmetry condition.

We will transform boundary-value problem (6)-(11), (16) to dimensionless variables:

$$\bar{r} = r\gamma_s/V\varepsilon_s,\ \bar{x} = x\gamma_s/V\varepsilon_s,\ \bar{\sigma}_{ik} = \sigma_{ik}/\sigma_s,$$
$$\bar{\varepsilon}_{ik} = \varepsilon_{ik}/\varepsilon_s\ (\bar{J} = J/\sigma_s).$$

We designate $\delta = \sigma_S/\tau_S$ $(0 \le \delta \le \infty)$. Here, the main relations (8)-(11) take the form

$$\bar{\varepsilon}_r = -\bar{\varepsilon}_\theta = \frac{1}{2}\bar{J}^{\varkappa}(\bar{\sigma}_r - \bar{\sigma}_\theta) + \frac{1}{2}\delta^{\lambda+1}\int\limits_{-\infty}^{\bar{x}} \bar{J}(\bar{\sigma}_r - \bar{\sigma}_\theta)\,d\bar{x}, \quad (17)$$

$$\bar{\varepsilon}_{r\theta} = \bar{J}^{\varkappa}\bar{\tau}_{r\theta} + \delta^{\lambda+1}\int\limits_{-\infty}^{\bar{x}} \bar{J}\bar{\tau}_{r\theta}\,d\bar{x},$$

while Eqs. (6) and (7) and conditions (16) remain the same (in new variables).

Thus, the solution of the boundary-value problem depends on three free parameters: δ, κ and λ. Computer calculations performed by numerical methods in the two limiting cases yield similarity asymptotes which are easily studied. We will examine them here.

At $\delta \to 0$, the "viscous" term in (17) is negligible, while the boundary-value problem permits the group of transformations $r' = c_1 r$, $\sigma_{ik}' = c_2\sigma_{ik}$, $\varepsilon_{ik}' = c_3\varepsilon_{ik}$, where c_1, c_2, c_3 are parameters of the group $(c_3 = c_2^{\kappa+1})$. It follows from this and from the finite value of Γ in Eq. (15) that the solution of the boundary-value problem has the form

$$\sigma_{ik} = \Omega_{ik}(\theta)\bar{r}^{\beta},\ \bar{\varepsilon}_{ik} = E_{ik}(\theta)\bar{r}^{-1-\beta}. \quad (18)$$

Inserting the solution (18) into (17) at $\delta = 0$ gives us [6, 25, 26] $\beta = -1/(\kappa + 2)$. Thus, at $\delta \to 0$

$$\sigma_{ik} = \sigma_s\Omega_{ik}(\theta)\left(\frac{\gamma_s r}{V\varepsilon_s}\right)^{-1/(\varkappa+2)},\quad \varepsilon_{ik} = \varepsilon_s E_{ik}(\theta)\left(\frac{\gamma_s r}{V\varepsilon_s}\right)^{-(\varkappa+1)/(\varkappa+2)}.$$

The functions Ω_{ik} and E_{ik} are determined numerically from the solution of the homogeneous boundary-value problem to within one arbitrary constant.

At $\delta \to \infty$, the "elastic" term in Eqs. (17) is negligible and the boundary-value problem permits the same group of transformations. Thus, its solution has the form (18). Inserting (18) into (17) at $\delta \gg 1$, it is not hard to see that $\beta = -2/(\lambda + 2)$. This means that at $\delta \to \infty$

$$\sigma_{ik} = \sigma_s\Omega_{ik}'(\theta)\left(\frac{\gamma_s r}{V\varepsilon_s}\right)^{-2/(\lambda+2)},\quad \varepsilon_{ik} = \varepsilon_s E_{ik}'(\theta)\left(\frac{\gamma_s r}{V\varepsilon_s}\right)^{-\lambda/(\lambda+2)}.$$

The functions Ω_{ik}' and E_{ik}' are determined numerically from the solution of the corresponding homogeneous boundary-value problem to within one arbitrary constant.

With arbitrary δ, the solution of the boundary-value problem is conveniently "joined together" from these two boundary-layer asymptotes in the following manner:

$$\bar{\sigma}_{ik} = A_{ik}(\theta, \delta)\bar{r}^{-1/(\varkappa+2)} + B_{ik}(\theta, \delta)\bar{r}^{-2/(\lambda+2)}, \quad (19)$$

where $A_{ik} \to \Omega_{ik}(\theta)$ at $\delta \to 0$; $B_{ik} \to \Omega_{ik}'(\theta)$ at $\delta \to \infty$. It is evident from this that if $\lambda > 2(\kappa + 1)$, then at $r \to 0$ the dominant term in solution (19) will be the first term, while at $r \to \infty$ it will be the second term; if $\lambda < 2(\kappa + 1)$, the the second term will be the dominant term in solution (19) at $r \to 0$ and the first term will be so at $r \to \infty$.

This means that at $\lambda > 2(\kappa + 1)$ there will be a small "plastic core" in the immediate vicinity of the tip of a moving crack. This core will be immersed in a "viscous" medium, and at $\lambda < 2(\kappa + 1)$ there exists a small viscous region near the tip of a moving crack immersed in a plastic medium. At $\lambda > 2(\kappa + 1)$, the medium will behave similarly to a viscous fluid, and its plasticity properties will be manifest only near the crack front (where they are dominant). At $\lambda < 2(\kappa + 1)$, the medium will behave similarly to an elastic body, and its viscous properties will be manifest only near the crack front (where they are dominant). At the branch point $\lambda = 2(\kappa + 1)$, the boundary-value problem permits a similarity solution for all $\bar{r}$, and the plasticity and creep properties of the medium are of equal importance for the motion of the crack.

The motion of the crack in the continuum is accompanied by the formation of a "core" near its front. This core has special properties compared to the properties of the material away from the front. The radius of the core is on the order of $V\varepsilon_s/\gamma_s$. At $\lambda > 2(\kappa + 1)$, the material of the core behaves in a nonlinearly elastic manner (while the material away from the front is similar to a nonlinearly viscous fluid), but at $\lambda < 2(\kappa + 1)$, the core material behaves in a nonlinearly viscous manner (while the material away from the front is similar to a nonlinearly elastic body).

We are grateful to V. G. Lagutin and A. L. Sadovnikov for their help in the survey at the introduction of the present study.

LITERATURE CITED

1. Yu. N. Rabotnov, Creep of Structural Elements [in Russian], Nauka, Moscow (1966).
2. G. P. Cherepanov, "Formation and growth of cracks in solids under creep conditions," in: Nonlinear Models and Problems of the Mechanics of Deformable Solids [in Russian], Nauka, Moscow (1984).
3. G. P. Cherepanov, "Point defects in solids," in: Fundamentals of Deformation and Fracture (Eshelby memorial volume), Univ. Press, Cambridge (1985).
4. M. J. Siverns and A. T. Price, "Crack growth under creep conditions," Nature, 228, No. 5273 (1970).
5. G. P. Irwin, Fracture. Handbuch der Physik, G, Springer-Verlag, Berlin (1958).
6. G. P. Cherepanov, "Crack propagation in continua," Prikl. Mat. Mekh., 31, No. 3 (1967).
7. J. R. Rice, "A path independent integral and the approximate analysis of strain concentration by notches and cracks," J. Appl. Mech., 35, No. 2 (1968).
8. M. L. Williams, "On the mathematical criterion for fracture," in: Thin-Shell Structures. Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1974).
9. J. D. Landes and J. A. Begley, "The I-integral as a fracture criterion," in: Fracture Toughness, Am. Soc. Test Mater. Spec. Tech. Publ., No. 514, Philadelphia (1972).
10. J. D. Landes and J. A. Begley, "A fracture mechanics approach to creep crack growth," in: Mech. Crack Growth Am. Soc. Test Mater. Spec. Tech. Publ., No. 590, Philadelphia (1976).
11. G. P. Cherepanov, Mechanics of Brittle Fracture, McGraw-Hill, New York (1979).
12. S. A. Shesterikov and A. M. Lokoshchenko, "Creep and rupture strength of metals. The mechanics of deformable solids," Itogi Nauki Tekh., 13 (1980).
13. M. F. Ashby and B. Tomkins, "Micromechanisms of fracture and elevated temperature fracture mechanics," in: Mechanical Behavior of Materials: Proc. 3rd Int. Conf., Cambridge, 1979. Univ. Press, Cambridge (1980), Vol. 1.
14. E. G. Ellision and M. P. Harper, "Creep behavior of components containing cracks. A critical review," J. Strain Anal., 13, No. 1 (1978).
15. L. S. Fu, "Creep crack growth in technical alloys at elevated temperature. A review," Eng. Fract. Mech., 13, No. 2 (1980).
16. H. P. Leeuwen, "The application of fracture mechanics to creep crack growth," Eng. Fract. Mech., 9, No. 4 (1977).
17. R. Pilkington, "Creep crack growth in low-alloy steels. Critical assessment," Met. Sci., 13, No. 10 (1979).
18. K. Sadamanda and P. Shahinian, "Review on the fracture mechanics approach to creep crack growth in structural alloys," Eng. Fract. Mech., 15, Nos. 3-4 (1981).
19. G. P. Cherepanov, "New crack-tip models. Fracture control of engineering structures," Proc. ECF — 6, Amsterdam. Eng. Mater. Advisory Services Ltd., Warley (1986).
20. A. A. Kaminskii, Fracture Mechanics of Viscoelastic Bodies [in Russian], Naukova Dumka, Kiev (1980).
21. C. Atkinson and J. D. Eshelby, "The flow of energy into the tip of a moving crack," Int. J. Fract. Mech., 4, No. 1 (1968).
22. E. N. Sher, "Energy condition at the tip of a moving crack," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1969).
23. G. P. Cherepanov, "Cracks in solids," Int. J. Solids Struct., 4, No. 4 (1968).
24. B. V. Kostrov, L. V. Nikitin, and L. M. Flitman, "Brittle fracture mechanics," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 3 (1969).

25. J. W. Hutchinson, "Singular behavior at the end of tensile crack in a hardening material," J. Mech. Phys. Solids, 16, No. 1 (1968).
26. J. R. Rice and G. F. Rosengren, "Plane strain deformation near a crack tip in a power-law hardening material," J. Mech. Phys. Solids, 16, No. 1 (1968).
27. A. A. Chizhik, "Crack resistance of high-temperature steels and alloys during creep," Fiz. Khim. Mekh. Mater., No. 1 (1986).
28. V. V. Bolotin, "Griffith crack in a damaged viscoelastic medium," Raschety Prochn., Mashinostroenie, No. 26 (1985).
29. L. V. Nikitin, "Application of the Griffith's approach to analysis of rupture in viscoelastic bodies," Int. J. Fract., 24, No. 2 (1984).
30. H. Riedel and J. R. Rice, "Tensile cracks in creeping solids," in: Fracture Mechanics, Am. Soc. Test. Mater. Spec. Tech. Publ., No. 700, Philadelphia (1980).
31. C. Y. Hui and H. Riedel, "The asymptotic stress and strain field near the tip of a growing crack under creep conditions," Int. J. Fract., 17, No. 4 (1981).
32. D. R. Hayhurst, P. R. Brown, and C. J. Morrison, "The role of continuum damage in creep crack growth," Philos. Trans. R. Soc. London, A311 (1984).
33. V. I. Astaf'ev, "Laws of crack growth under creep conditions," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 1 (1986).
34. V. I. Astaf'ev, "Description of the fracture process under creep conditions," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1986).
35. L. N. McCartney, "On the energy balance approach to fracture in creeping materials," Int. J. Fract., 19, No. 1 (1982).
36. V. A. Kiselev, "Analysis of crack propagation under creep conditions," Probl. Prochn., No. 4 (1983).
37. G. P. Cherepanov, "Initiation of microcracks and dislocations," Prikl. Mekh., No. 12 (1987).
38. G. P. Cherepanov, "Microcrack growth under monotonic loading," Prikl. Mekh., No. 4 (1988).
39. G. P. Cherepanov, "Closing of microcracks during unloading and the formation of reverse dislocations," Prikl. Mekh., No. 7 (1988).
40. G. P. Cherepanov, "Current problems of fracture mechanics," Probl. Prochn., No. 10 (1987).

NUMERICAL INVESTIGATION OF THE PROCESS OF NONDEFORMABLE CYLINDER PENETRATION AT CONSTANT VELOCITY INTO A COMPRESSIBLE FLUID

S. M. Bakhrakh, O. A. Vinokurov, G. V. Gorbenko,
N. P. Kovalev, Yu. A. Osipov, and T. A. Toropova

UDC 531.66

Detailed investigation of the process of nondeformable solid penetration into different media is of great interest in connection with a number of scientific-technical problems of practical importance. Analytical, experimental, and numerical methods (see [1-3, 4-6, 7-10], respectively, say) are utilized to solve the problems occurring here. Because of the complexity of solving the problems by an analytical method, the analysis of a limited number of situations turns out to be accessible. Formulation of experiments in this area is fraught with a number of difficulties. Moreover, the integrated characteristics of the process, for instance, the depth of penetration of the body, are usually fixed in the experiments. A detailed pattern of impactor interaction with deformable compressible media can be obtained by using the numerical solution of similar problems.

The process of bodies of cylindrical shape penetrating a compressible fluid is investigated in this paper by numerical modeling methods. Dependences of the main characteristics of the process (the drag force F, the cavern location relative to the body) on the Mach number $M = V/c_0$ (V is the insertion velocity, and c_0 is the sound speed in the obstacle

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 150-155, September-October, 1989. Original article submitted April 27, 1988.

0021-8944/89/3005-0815$12.50